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Book Review: *Chaotic Transitions in Deterministic and Stochastic Dynamical Systems*

Chaotic Transitions in Deterministic and Stochastic Dynamical Systems. Emil Simiu, 224 pp., Princeton University Press, 2002

Consider an elastic metallic beam suspended from a rigid frame over two magnets, equidistant from the undeformed position of the beam. The frame and magnets are fixed to a table that can be submitted to controlled periodic horizontal shaking. Depending on the amplitude and frequency of the shaking, there result three types of steady-state motion: (a) the beam oscillates periodically about one of the magnets, (b) oscillates periodically between the two magnets, or (c) meanders erratically in a *chaotic* sort of motion, punctuated by irregular visits to each of the equilibria. With this simple, engaging example Emil Simiu introduces us to the topic of his latest book, *Chaotic Transitions in Deterministic and Stochastic Dynamical Systems*.

Published in the new "Princeton Series in Applied Math," the book addresses the question of transitions between preferred regions of phase space in planar systems, that is, systems that are well described by two variables (such as the displacement and velocity of the tip of the beam in the introductory example). A unified approach for the study of such transitions is presented in the form of Melnikov's analysis. Simple zeros of the Melnikov function—a measure of the distance between the stable and unstable manifolds in planar systems—are a necessary condition for *homoclinic chaos*, or chaotic transitions between the various centers of attraction. A main theme of the book is the equivalence between deterministic and stochastic systems (systems with stochastic input), which is nicely explained within the framework of Melnikov theory.

The book is largely self-contained, accessible to beginner graduate students in the exact sciences, and is intended for a wide readership in the fields of mathematics, physics and engineering. In the first part, prerequisite concepts (planar integrable systems, deterministic Melnikov theory, basics of stochastic processes) are introduced, followed by the development

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of Melnikov's method to stochastic planar systems. The second part focuses on applications, including stochastic resonance, nonlinear control, vessel roll dynamics, buckling columns, and interpretation of experiments in auditory nerves.

A particularly welcome feature of the book is that it gathers diverse concepts useful in the study of nonlinear dynamics and chaotic systems, in one concise, elegant publication. Apart from the basic definitions of flow and fixed points, homoclinic and heteroclinic orbits, manifolds, and Poincaré maps, used in presenting the Melnikov theory for deterministic systems, part I includes an introduction to chaos (sensitivity to initial conditions, Lyapunov exponents, attractors), and a clear exposition of the Smale horseshoe map and the shift map, symbolic dynamics and the space Σ_2, culminating with the Smale–Birkhoff theorem, required to establish the necessary conditions for homoclinic chaos. The discussion of transitions in stochastic dynamical systems is preceded with an introduction to elementary concepts in stochastic processes, such as spectral density, covariance, and filters.

The brevity of the text is also its bane. The dreaded "it can be shown" is invoked sparingly (and mostly in the appendices), though a student might feel more comfortable with additional intermediate computational steps elsewhere throughout the text, or with more elaboration on the physical meaning of the various examples. But, in my opinion, this is a small price to pay for the highly readable, elegant, and concise result achieved by the author. The book is well referenced, handsomely making up for many of its omissions, and relatively free of typographical errors (a rare, unfortunate misprint occurs in the sign of the dissipative force, in the first concrete example of an application of Melnikov's theory, on p. 33).

Emil Simiu has succeeded in putting together a highly stimulating book that proposes a promising, unifying approach to various aspects of chaos theory. While encompassing a wide swath of topics, traditionally found only on scattered sources, the book is succinctly written, exhibiting a quality reserved to the best of review works. The book is sure to find a place of honor on the shelves of researchers in the fields of dynamical systems, stochastic processes, and stochastic differential equations.

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